GATE 2012

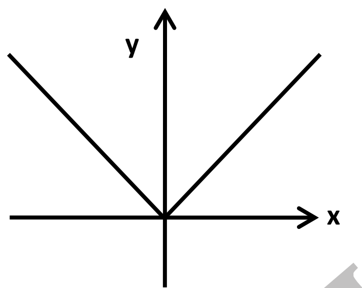
Mechanical Engineering

Set - C

Q. 1 – Q. 25 carry one mark each.

1. Consider the function $f(x) = |x|$ in the interval $-1 \le x \le 1$. At the point $x = 0, f(x)$ is
(A) Continuous and differentiable.
(B) Non-continuous and differentiable
(C) Continuous and non-differentiable
(D) Neither continuous nor differentiable

[Ans. C]
$f(x) = |x| -1 \le x \le 1$



at x = 0
$\lim_{x\to 0} + f(x) = \lim_{x\to 0} - f(x) = \lim_{x\to 0} + f(x)$
∴ It is continuous.
$\lim_{x\to 0} + \frac{f(x+h) - f(x)}{h} \neq \lim_{x\to 0} - \frac{f(x) - f(x-h)}{h}$
∴ It is not differentiable.

2. $\lim_{x\to 0} \left(\frac{1-cosx}{x^2}\right)$ is
(A) 1/4
(B) 1/2
(C) 1
(D) 2

[Ans. B]
$\lim_{x\to 0} + \frac{1-\cos x}{x^2}$

$$= \lim_{x\to 0} \frac{1 - 1 + 2\sin^2 \frac{x}{2}}{x^2}$$

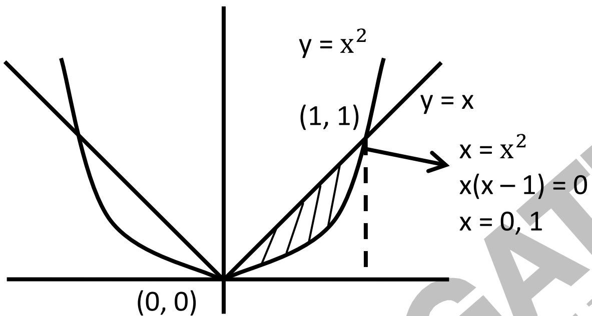
$$= \lim_{x\to 0} \frac{1}{2} \times \frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2}$$

$$= \frac{1}{2}$$

3. The area enclosed between the straight line y = x and the parabola $y = x^2$ in the x-y plane is

(A) 1/6
(B) 1/4
(C) 1/3
(D) 1/2

[Ans. A]



Area enclosed between y = x & $y = x^2$ is

$= \int_0^1 y_1 dx - \int_0^1 y_2 dx$

$= \int_0^1 x\, dx - \int_0^1 x^2 dx$

$= \left(\frac{x^2}{2}\right)_0^1 - \left(\frac{x^3}{3}\right)_0^1$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$

4. A thin walled spherical shell is subjected to an internal pressure. If the radius of the shell is increased by 1% and the thickness is reduced by 1%, with the internal pressure remaining the same, the percentage change in the circumferential (hoop) stress is

(A) 0
(B) 1
(C) 1.08
(D) 2.02

[Ans. D]

Hoop stress = $\frac{Pd}{4t} = \frac{Pr}{2t}$

% change in Hoop stress

$= \frac{\left(\frac{P \times 1.01r}{2 \times 0.99r} - \frac{Pr}{2t}\right)}{\frac{Pr}{2t}} \times 100$

= (1.0202 – 1) × 100

= 2.02%

5. A solid disc of radius *r* rolls without slipping on a horizontal floor with angular velocity ω and angular acceleration α. The magnitude of the acceleration of the point of contact on the disc is

(A) Zero

(B) rα

(C) $\sqrt{(r\alpha)^2 + (r\omega^2)^2}$

(D) $r\omega^2$

[Ans. D]

Tangential acceleration will be zero, radial acceleration will exist.

Hence, Total acceleration = $r\omega^2$

6. The following are the data for two crossed helical gears used for speed reduction:

Gear I : Pitch circle diameter in the plane of rotation 80 mm and helix angle 30°

Gear II : Pitch circle diameter in the plane of rotation 120 mm and helix angle 22.5°

If the input speed is 1440 rpm, the output speed in rpm is

(A) 1200

(B) 900

(C) 875

(D) 720

[Ans. B]

For helical gears

Velocity ratio = $\frac{d_1 \cos\varphi}{d_2 \cos\varphi} = \frac{80 \cos 30^o}{120 \cos 22.5} = \frac{N_2}{N_1}$

N_2 = 1440 × 0.625 = 900 rpm

7. Steam enters an adiabatic turbine operating at steady state with an enthalpy of 3251.0 kJ/kg and leaves as a saturated mixture at 15 kPa with quality (dryness fraction) 0.9. The enthalpies of the saturated liquid and vapor at 15 kPa are $h_f = 225.94$ kJ/kg and $h_g = 2598.3$ kJ/kg respectively. The mass flow rate of steam is 10 kg/s. Kinetic and potential energy changes are negligible. The power output of the turbine in MW is

(A) 6.5

(B) 8.9

(C) 9.1

(D) 27.0

[Ans. B]

$h_1 = 3251.0$ kJ/kg.

$h_2 = h_1 + x$. kg.

$h_2 = 225.94 + 0.9 \times (2598.3 - 225.99)$

$h_2 = 2361.06$ kJ/kg.

m = 10 kg/S.

Power $= m(h_1 - h_2)$

$= 10 \times (3251 - 2361.06)$

$= 8899.36$ kW

$= 8.9$ MW.

8. For an opaque surface, the absorptivity (α), transmissivity (τ) and reflectivity (ρ) are related by the equation:

(A) $\alpha + \rho = \tau$
(C) $\alpha + \rho = 1$
(B) $\rho + \alpha + \tau = 0$
(D) $\alpha + \rho = 0$

[Ans. C]

For opaque surface,

$\alpha + \rho = 1$.

9. Oil flows through a 200 mm diameter horizontal cast iron pipe (friction factor, $f = 0.0225$) of length 500 m. The volumetric flow rate is 0.2 m^3/s. The head loss (in *m*) due to friction is (assume $g = 9.81$ m/s^2)

(A) 116.18
(C) 18.22
(B) 0.116
(D) 232.36

[Ans. A]

4f = 0.0225

L = 500 m

Q = 0.2 m^3/S

d = 200 mm , $V = \frac{Q}{\frac{\pi}{4}d^2} =$

$$h_f = \frac{4f\,LV^2}{2gd}$$

$$= \frac{0.225 \times 500 \times V^2}{2 \times 9.81 \times 0.2}$$

$h_f = 116.18$ m.

10. During normalizing process of steel, the specimen is heated
(A) Between the upper and lower critical temperature and cooled in still air.
(B) Above the upper critical temperature and cooled in furnace.
(C) Above the upper critical temperature and cooled in still air.
(D) Between the upper and lower critical temperature and cooled in furnace

[Ans. C]
In normalization process
Steel is heated above upper critical temperature and cooled in still air.

11. In an interchangeable assembly, shafts of size $25.000^{+0.040}_{-0.010}$ mm mate with holes of size $25.000^{+0.030}_{-0.020}$ mm. The maximum interference (in macrons) in the assembly is
(A) 40
(B) 30
(C) 20
(D) 10

[Ans. C]
+ 0.040
– 0.010
Shaft size 25.000 mm
24 mm 90 μm < d_5 < 25 mm 40 μm
+ 0.030
+ 0.020
Hole size $25.000^{+0.030}_{+0.020}$
25 mm 20 μm < d_h < 25 mm 30 μm
Max Interference
= Max shaft size – Min Hole size
= 25 mm 40 μm – 25 mm 20 μm
= 20 μm

12. Match the following metal forming processes with their associated stresses in the work-piece.

	Metal forming process		Type of stress
1.	Coining	P.	Tensile
2.	Wire Drawing	Q.	Shear
3.	Blanking	R.	Tensile and compressive
4.	Deep Drawing	S.	Compressive

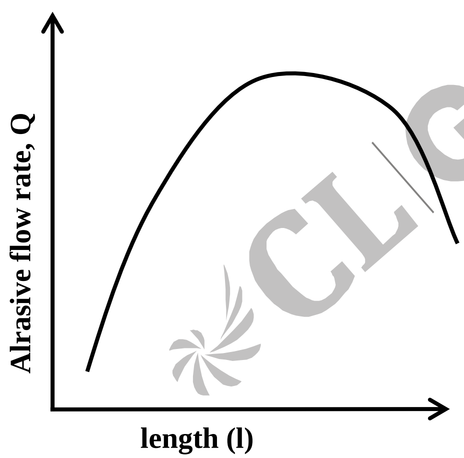
(A) 1-S, 2-P, 3-Q, 4-R
(B) 1-S, 2-P, 3-R, 4-Q
(C) 1-P, 2-Q, 3-S, 4-R
(D) 1-P, 2-R, 3-Q, 4-S

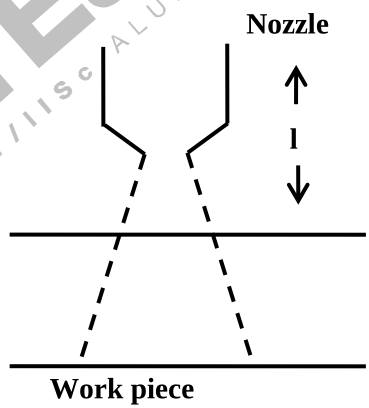
[Ans. A]

Coining $\rightarrow$ Compressive
Wire drawing $\rightarrow$ Tensile
Blanking $\rightarrow$ Shear
Deep Drawing $\rightarrow$ Tensile and Compressive

13. In abrasive jet machining, as the distance between the nozzle tip and the work surface increases, the material removal rate
(A) Increases continuously
(B) Decreases continuously
(C) Decreases, becomes stable and then increases
(D) Increases, becomes stable and then decreases

[Ans. D]





Graph of abrasive flow rate v/s tool tip distance metal removal rate is proportional to abrasive flow rate. As the tool tip distance increase, area of contact increases which results in higher metal removal rate. But after a certain distance, the jet looses its velocity, hence metal removal rate decreases.

14. For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by

(A) $\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}$

(B) $\frac{1}{\sqrt{2}}\,\hat{\imath} - \frac{1}{\sqrt{2}}\,\hat{\jmath}$

(C) $\hat{k}$

(D) $\frac{1}{\sqrt{3}}\,\hat{\imath} + \frac{1}{\sqrt{3}}\,\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

[Ans. A]

To find the direction of normal, take the gradient i.e.

$\overline{\nabla} \cdot f(x)$

Where $f(x) = x^2 + y^2 + 2^2 - 1$

Now $\overline{\nabla} \cdot f(x) = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$

So at point $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

$\vec{N} = v = \vec{i} + \sqrt{2}\vec{j} + 0 \cdot \vec{k}$

8 unit vector will become

$$\frac{\overrightarrow{i}}{\sqrt{2}} + \frac{\overrightarrow{J}}{\sqrt{2}} + 0 \cdot \vec{k}$$

15. At x = 0, the function $f(x) = x^3 + 1$ has

(A) A maximum value

(B) A minimum value

(C) A singularity

(D) A point of inflection

[Ans. D]

At x = 0,

$$\left.\frac{df(x)}{dx}\right|_{x=0} = 0$$

$$\left.\frac{d^2f(x)}{dx^2}\right|_{x=0} = 6x|_{x=0} = 0 \;\&\; \left.\frac{d^3f(x)}{dx^3}\right|_{x=0} \neq 0$$

So, x = 0 is point of inflexion

16. For a long slender column of uniform cross section the ratio of critical buckling load for the case with both ends clamped to the case with both ends hinged is

(A) 1

(B) 2

(C) 4

(D) 8

[Ans. C]

Critical buckling load for both end clamped = $\frac{4\pi^2 EI}{L^2}$

Critical buckling load for both end hinged = $\frac{\pi^2 EI}{L^2}$

Ratio = 4

17. A cantilever beam of length L is subjected to a moment M at the free end. The moment of inertia of the beam cross section about the neutral axis is *l* and the Young's modulus is *E*. The magnitude of the maximum deflection is

(A) $\frac{ML^2}{2EI}$
(C) $\frac{2ML^2}{EI}$
(B) $\frac{ML^2}{EI}$
(D) $\frac{4ML^2}{EI}$

[Ans. A]

Max Deflection = $\frac{ML^2}{2EI}$

18. A circular solid disc of uniform thickness 20 mm, radius 200 mm and mass 20 kg, is used as a flywheel. If it rotates at 600 rpm, the kinetic energy of the flywheel, in *Joules* is

(A) 395
(C) 1580
(B) 790
(D) 3160

[Ans. B]

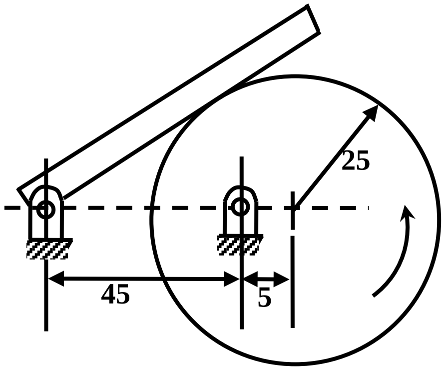
K.E. $= \frac{1}{2} I\, w^2$

$= \frac{1}{2} \times \frac{1}{2} Mr^2 \times \left(\frac{2\pi N}{60}\right)^2$

$= \frac{1}{2} \times \frac{1}{2} 20 \times (0.2)^2 \times (20\pi)^2$

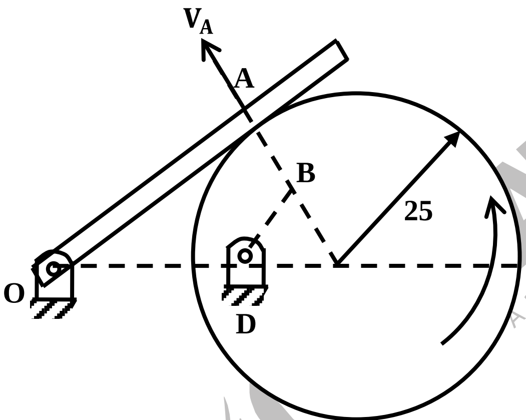
= 790 J.

19. In the mechanism given below, if the angular velocity of the eccentric circular disc is 1 rad/s, the angular velocity (rad/s) of the follower link for the instant shown in the figure is



(A) 0.05
(B) 0.1
(C) 5.0
(D) 10.0

[Ans. B]



Ac is ⊥ 0A ω = 1 rad/s
We draw ⊥ from D on AC
Δ OAC is similar to Δ DBC
OA = $\sqrt{50^2 - 25^2}$ = 43.3 mm
$\frac{OA}{DB} = \frac{OC}{DC} \Rightarrow \frac{43.3}{DB} = \frac{50}{5} \Rightarrow$DB = 43.3 mm
Velocity A = velocity of B
$V_A = V_B$ = DB × ω = 4.33 × 1 = 4.33
Angular velocity of OA, $\omega_{OA} = \frac{V_A}{OA} = \frac{4.33}{43.3}$ = 0.1 rad/s

20. An ideal gas of mass m and temperature T_1 undergoes a reversible isothermal process from an initial pressure P_1 to final pressure P_2. The heat loss during the process is Q. The entropy change ΔS of the gas is

(A) $mR \ln\left(\frac{P_2}{P_1}\right)$

(B) $mR \ln\left(\frac{P_1}{P_2}\right)$

(C) $mR \ln\left(\frac{P_2}{P_1}\right) - \frac{Q}{T_1}$

(D) Zero

[Ans. B]

For isothermal process, T = C

$\Delta S = \text{mR} \ln\left(\frac{\text{P}_1}{\text{P}_2}\right).$

21. Which one of the following configurations has the highest fin effectiveness?

(A) Thin, closely spaced fins

(B) Thin, widely spaced fins

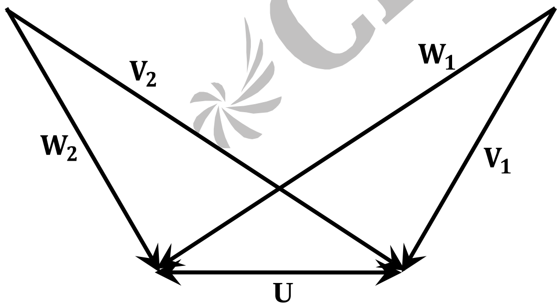
(C) Thick, widely spaced fins

(D) Thick, closely spaced fins

[Ans. A]

For a given configuration, Thin, closely spaced fins can achieve maximum heat transfer area. Hence, more effective.

22. The velocity triangles at the inlet and exit of the rotor of a turbo-machine are shown. V denotes the absolute velocity of the fluid, W denotes the relative velocity of the fluid and U denotes the blade velocity. Subscripts 1 and 2 refer to inlet and outlet respectively. If $V_2 = W_1$ and $V_1 = W_2$, then the degree of reaction is



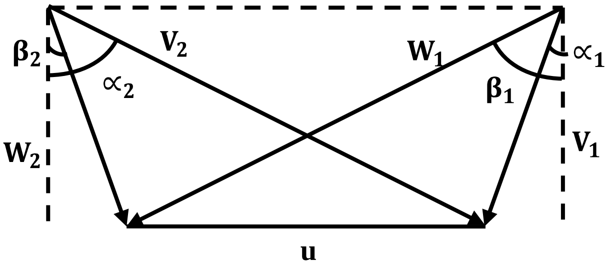
(A) 0

(B) 1

(C) 0.5

(D) 0.25

[Ans. C]



$$\left.\begin{array}{l} V_1 = W_2 \\ V_2 = W_1 \end{array}\right\}$$

V → absolute velocity

W → relative velocity

$\propto_1$ → inlet nozzle angle

β_1 → inlet blade angle

$\propto_2$ → outlet angle → inlet to stator angle

β_2 → outlet blade angle

Triangles are similar,

$$\left.\begin{array}{l} \propto_1 = \beta_2 \\ \propto_2 = \beta_1 \end{array}\right\}$$

Thus Degree of Reaction R = 0.5 = $\frac{1}{2}$

23. A solid cylinder of diameter 100 mm and height 50 mm is forged between two frictionless flat dies to a height of 25 mm. The percentage change in diameter is

(A) 0
(B) 2.07
(C) 20.7
(D) 41.4

[Ans. D]

$\pi D_1^2 L_1 = \pi D_2^2 L_2$

$(100)^2 \times 50 = D_2^2 \times 25$

$D_2 = 100\sqrt{2}$

% change = $\frac{D_2 - D_1}{D_1} = \frac{100\sqrt{2} - 100}{100} \times 100$

= 41.4 %

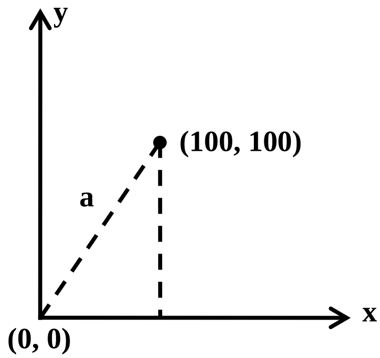
24. A CNC vertical milling machine has to cut a straight slot of 10 mm width and 2 mm depth by a cutter of 10 mm diameter between points (0, 0) and (100, 100) on the XY plane (dimensions in mm). The feed rate used for milling is 50 mm/min. Milling time for the slot (in seconds) is

(A) 120
(B) 170
(C) 180
(D) 240

[Ans. B]

Given data: width (b) = 10 mm

depth (t) = 2 mm



points (0, 0) & (100, 100)

Distance travelled = $\sqrt{100^2 + 100^2}$

$= 100\sqrt{2}$

Milling time $= \frac{100\sqrt{2}}{50}$ min = $\frac{100\sqrt{2}}{50}$x60 sec

$= 120\sqrt{2} = 170$ secs

25. Which one of the following is NOT a decision taken during the aggregate production planning stage?

(A) Scheduling of machines
(B) Amount of labour to be committed
(C) Rate at which production should happen
(D) Inventory to be carried forward

[Ans. A]

Scheduling of Machines.

Q. 26 To Q.55 carry two marks each.

26. For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized Eigen vectors is given as

(A) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$

(B) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(C) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$

(D) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

[Ans. B]

$A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$

To find eigenvector, first find Eigen-values by solving equation

$|\lambda I - A| = 0$

$\Rightarrow \begin{vmatrix} \lambda - 5 & -3 \\ -1 & \lambda - 3 \end{vmatrix} = 0$

$\Rightarrow \lambda = 6, 2$

So for λ = 2, the eigenvector 15

$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Normalized Eigen Vector

$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$

27. The inverse Laplace transform of the function $F(s) = \frac{1}{s(s+1)}$ is given by

(A) $f(t) = \sin t$

(B) $f(t) = e^{-t} \sin t$

(C) $f(t) = e^{-t}$

(D) $f(t) = 1 - e^{-t}$

[Ans. D]

$F(s) = \frac{1}{S(S+1)}$

$= \frac{1}{S} - \frac{1}{S+1}$

$\alpha^{-1}F(S) = (1 - e^{-t})\ 4(t)$

28. The state of stress at a point under plane stress condition is

$\sigma_{xx} = 40$ MPa, $\sigma_{yy} = 100$ MPa and $\tau_{xy} = 40$ MPa

The radius of the Mohr's circle representing the given state of stress in MPa is

(A) 40
(B) 50
(C) 60
(D) 100

[Ans. B]

$\sigma_{xx} = 40\text{MPa}$

$\sigma_{yy} = 100\text{MPa}$

$\tau_{xy} = 40\text{MPa}$

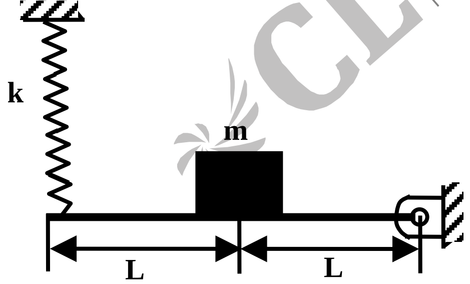
Radios of Mohr's circle

$$= \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

$$= \sqrt{\left(\frac{100 - 40}{2}\right)^2 + (40)^2}$$

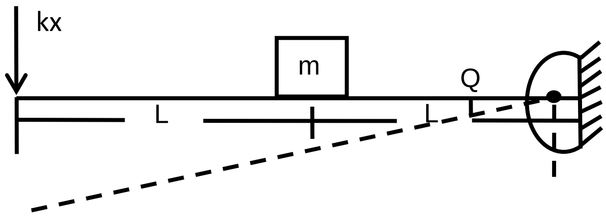
= 50 MPa

29. A concentrated mass *m* is attached at the centre of a rod of length 2L as shown in the figure. The rod is kept in a horizontal equilibrium position by a spring of stiffness *k*. For very small amplitude of vibration, neglecting the weights of the rod and spring, the undamped natural frequency of the system is



(A) $\sqrt{\frac{k}{m}}$
(B) $\sqrt{\frac{2k}{m}}$
(C) $\sqrt{\frac{k}{2m}}$
(D) $\sqrt{\frac{4k}{m}}$

[Ans. D]



Equation of motion.

$m\ddot{x}.L + kx \times 2L = 0$

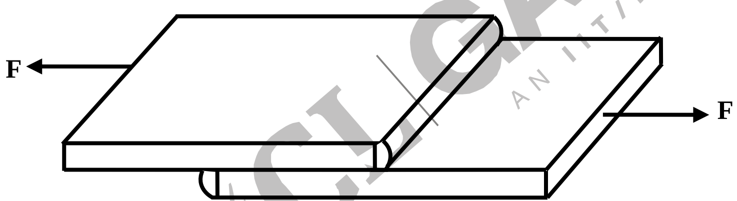
$m(L\ddot{\theta}).L\overline{\theta} + k(2L.\theta) \times 2L = 0$

$mL^2\ddot{\theta} + 4kL^2Q = 0$

$\ddot{\theta} + \frac{4k}{m}\theta = 0$

$w_n = \sqrt{\frac{4k}{m}}$

30. A fillet welded joint is subjected to transverse loading F as shown in the figure. Both legs of the fillets are of 10 mm size and the weld length is 30 mm. If the allowable shear stress of the weld is 94 MPa, considering the minimum throat area of the weld, the maximum allowable transverse load in kN is



(A) 14.44
(B) 17.92
(C) 19.93
(D) 22.16

[Ans. C]

S = 10 mm.

L = 30 mm.

$\tau_f = 94\ MPa$

$P = 0.707 \times 4 \times L \times S$

= 0.707 × 94 × 30 × 10

= 19.93 kN

31. A room contains 35 kg of dry air and 0.5 kg of water vapor. The total pressure and temperature of air in the room are 100 kPa and 25℃ respectively. Given that the saturation pressure for water at 25℃ is 3.17 kPa, the relative humidity of the air in the room is

(A) 67%
(B) 55%
(C) 83%
(D) 71%

[Ans. D]

Absolute Humidity = $\omega = \frac{0.5}{3.5} = \frac{mv}{mg} = 0.142857$

P_g = 3.17 KPa → Saturation pressure of water @ 25

P = 100 KPa → total pressure.

ϕ(R.H .) $\Rightarrow \frac{0.622\phi P_g}{P - \phi P_g} = \omega$

Or, $\phi = \frac{\omega P}{(\omega + 0.622)\, P_g} = \frac{0.1428 \times 100}{(0.1428 + 0.622) \times 3.17} = 0.708$

ϕ = 71%

32. An incompressible fluid flows over a flat plate with zero pressure gradients. The boundary layer thickness is 1 mm at a location where the Reynolds number is 1000. If the velocity of the fluid alone is increased by a factor of 4, then the boundary layer thickness at the same location, in mm will be

(A) 4
(B) 2
(C) 0.5
(D) 0.25

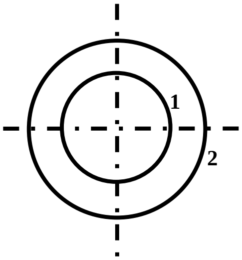
[Ans. C]

Boundary layer thickness (δ) $\alpha \frac{1}{\sqrt{R_e}}$

$R_e \,\alpha\, V \Rightarrow \delta \,\alpha\, \frac{1}{\sqrt{V}}$

If velocity of fluid is increased by 4 times then boundary layer thickness reduces by half

33. Consider two infinitely long *thin* concentric tubes of circular cross section as shown in the figure. If D_1 and D_2 are the diameters of the inner and outer tubes respectively, then the view factor F_{22} is given by



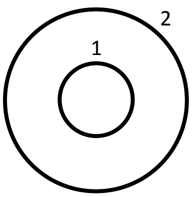
(A) $\left(\frac{D_2}{D_1}\right) - 1$

(B) $zero$

(C) $\left(\frac{D_1}{D_2}\right)$

(D) $1 - \left(\frac{D_1}{D_2}\right)$

[Ans. D]



$F_{11} = 0$

$F_{11} + F_{12} = 1 \Rightarrow F_{12} = 1$

$A_1 + F_{12} = A_2 F_{21}$

$\Rightarrow F_{21} = \frac{A_1}{A_2} F_{12}$

$\Rightarrow F_{21} = \frac{\Pi P_1 L}{\Pi P_2 L} \times 1$

$\Rightarrow F_{21} = \frac{D_1}{D_2}$

$F_{21} + F_{22} = 1$

$\Rightarrow F_{22} = 1 - \frac{D_1}{D_2}$

34. In a single pass drilling operation, a through hole is 15 mm diameter is to be drilled in a steel plate of 50 mm thickness. Drill spindle speed is 500 rpm, feed is 0.2 mm/rev and drill point angle is 118°. Assuming 2 mm clearance at approach and exit, the total drill time (in seconds) is

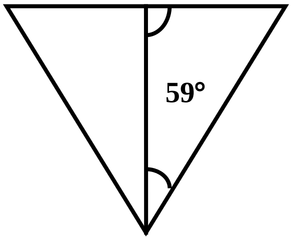
(A) 35.1

(B) 32.4

(C) 31.2

(D) 30.1

[Ans. A]



$tan\ 59 = \frac{7.5}{x}$

$x = 4.5$

$f = 0.2\ mm/rev$

$N = 500\ rpm$

$t = 50\ mm$

$d = 15\ mm$

Total travel $= 4.5 + 2 + 2 + 50$

$= 58.5$

$Time = \frac{Total\ travel}{Spindle\ speed}$

$= \frac{58.5}{0.2 \times 500}$ min

$= 35.1\ sec.$

35. Details pertaining to an orthogonal metal cutting process are given below.

Chip thickness ratio	0.4
Undeformed thickness	0.6 mm
Rake angle	+10°
Cutting speed	2.5 m/s
Mean thickness of primary shear zone	25 microns

The shear strain rate in s^{-1} during the process is

(A) 0.1781×10^5
(B) 0.7754×10^5
(C) 1.0104×10^5
(D) 4.397×10^5

[Ans. C]

$r = 0.4$

$t_1 = 0.6$ mm

$\alpha = 10^o$

$V_C = 2.5 m/s$

$t_m = 25\mu m$

$\tan\phi = \frac{r\cos\alpha}{1-r\sin\alpha} = 0.4233 \Rightarrow \phi = 22.9^o$

Shear strain rate $= \frac{V_C\cos\alpha}{\cos(\phi-\alpha)\times t_m} = \frac{2.5\times\cos 10}{\cos 12.9\times 25\times 10^{-6}} = 1.010^4\times 10^5/S$

36. The homogeneous state of stress for a metal part undergoing plastic deformation is

$$T = \begin{pmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{pmatrix}$$

Where the stress component values are in MPa. Using von Mises yield criterion, the value of estimated shear yield stress, in MPa is

(A) 9.50
(B) 16.07
(C) 28.52
(D) 49.41

[Ans. B]

$\sigma_{eq} = \sqrt{\frac{1}{2}\{(\sigma_{11}-\sigma_{22})^2 + (\sigma_{11}-\sigma_{22})^2 + 6[\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2]\}}$

We know, σ_{11}= 10, σ_{22} = 20, σ_{33} = −10; σ_{12} = 5; $\sigma_{23} = \sigma_{13}$ = 0

∴ σ_{eq} = 27.839 MPa

Shear stress at yield $\tau_y = \frac{\sigma_{eq}}{\sqrt{3}}$ = 16.07 Mpa

37. $+2y + z = 4$

$2x + y + 2z = 5$

$x - y + z = 1$

The system of algebraic equation given above has

(A) A unique solution of $x = 1, y = 1$ and $z = 1$.
(B) Only the two solutions of $(x = 1, y = 1, z = 1)$ and $(x = 2, y = 1, z = 0)$.
(C) Infinite number of solutions
(D) No feasible solution

[Ans. C]

From Cramer's Rule

$|A| = 0$

⟹ infinite no of solutions exist

38. Consider the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$ with the boundary conditions of $y(0)=0$ and $y(1)=1$. The complete solution of the differential equation is

(A) x^2

(B) $\sin\left(\frac{\pi x}{2}\right)$

(C) $e^x\sin\left(\frac{\pi x}{2}\right)$

(D) $e^{-x}\sin\left(\frac{\pi x}{2}\right)$

[Ans. A]

Quick Solution:

If we look of option (C) and (D) these doesn't satisfy boundary condition.

Out of options (B) & (C), (B) doesn't satisfy the differential equation itself. So correct option is (A)

Detailed Solution:

Given D.E. of the form

$x^2\, D^2y + xDy - 4y = 0 \qquad \left(\text{where } D = \frac{d}{dx}\right)$

For such type of equation (i.e. $(xD)^k$) use standard substitution

$y = x^k$

$\therefore Dy = kx^{k-1}$; $D^2y = k(k-1)\, x^{k-2}$

$\therefore x^2\, k(k-1)\, x^{k-2} + xk\, x^{k-1} - 4x^k = 0$

$x^k[k(k-1) + k-4) = 0$

as $x^k \neq 0 \qquad \therefore k\,(k-1) + k - 4 = 0$

$k^2 - k + k - 4 = 0$

$\therefore k^2 - 4 = 0 \qquad \therefore k = 2, -2$

In general $y = ax^2 + a_2\, x^{-2} + b$.

$y\,(0) = 0$

$0 = a_1\,(0) + \frac{a_2}{0} + b$

Not feasible as value tends to ∞

$\therefore$ k should be 2

$0 = 0 + b$

$\therefore b = 0$

$\therefore\ y = x^2$

$y\,(1) = 1$

$1 = a_1\,(1) + b$

$\therefore a_1 + b = 1$

$\therefore\ a_1 = 1$

39. A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is

(A) 1/20
(B) 1/12
(C) 3/10
(D) 1/2

[Ans. D]

The number of chances that the event will be successful is

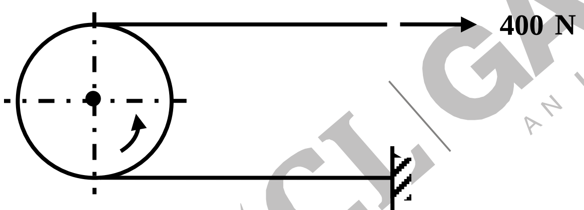
$$4_{C_1} \times 6_{C_2} = \frac{4 \times \angle 6}{\angle 2\,\angle 4}$$

Total number of chances for sample space is

$$6\ {}^{10}C_3 = \frac{\angle 10}{\angle 3 \angle 7}$$

So, probability is $\frac{\frac{4\times 6.5}{2}}{\frac{10.98}{3.2}} = \frac{1}{2}$

40. A force of 400 N is applied to the brake drum of 0.5 m diameter in a band-brake system as shown in the figure, where the wrapping angle is 180°. If the coefficient of friction between the drum and the band is 0.25, the braking torque applied, in N.m is



(A) 100.6
(B) 54.4
(C) 22.1
(D) 15.7

[Ans. B]

$T_1 = 400N$

$\mu = 0.25$

θ = 180°

= π radian.

d = 0.5 m, $r = \frac{0.5}{2} = 0.25m$

$$T_2 = \frac{T_1}{e^{\mu\theta}}$$

$T_2 = \frac{400}{e^{(0.25\times\pi)}} = 182.37$

Braking Torque $= (T_1 - T_2).r$

= (400 – 182.37) × 0.25

= 54.4

41. A solid circular shaft needs to be designed to transmit a torque of 50 N.m. If the allowable shear stress of the material is 140 MPa, assuming a factor of safety of 2, the minimum allowable design diameter in *mm* is

(A) 8
(B) 16
(C) 24
(D) 32

[Ans. B]

T = 50 Nm

τ = 140 Mpa

IOS = 2

$\frac{\tau}{FOS} = \frac{16T}{\pi d^3}$

$d = 3\sqrt{\frac{16\times 50\times 1000}{70\times\pi}}$

d = 15.38 mm ~16mm.

42. A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α. Young's modulus is *E* and the Poissons's ratio is υ, the thermal stress developed in the cube due to heating is

(A) $-\frac{\alpha(\Delta T)E}{(1-2\upsilon)}$
(B) $-\frac{2\alpha(\Delta T)E}{(1-2\upsilon)}$
(C) $-\frac{3\alpha(\Delta T)E}{(1-2\upsilon)}$
(D) $-\frac{\alpha(\Delta T)E}{3(1-2\upsilon)}$

[Ans. A]

$$\left.\begin{aligned}\epsilon_x &= \frac{\sigma_x}{E} - \frac{v(\sigma_y+\sigma_z)}{E}\\ \epsilon_y &= \frac{\sigma_y}{E} - \frac{v(\sigma_x+\sigma_z)}{E}\\ \epsilon_z &= \frac{\sigma_z}{E} - \frac{v(\sigma_x+\sigma_y)}{E}\end{aligned}\right\}\text{ for the cube from symmetry } \epsilon_x = \epsilon_y = \epsilon_z$$

The thermal expansion constraint → compressive stress on the material

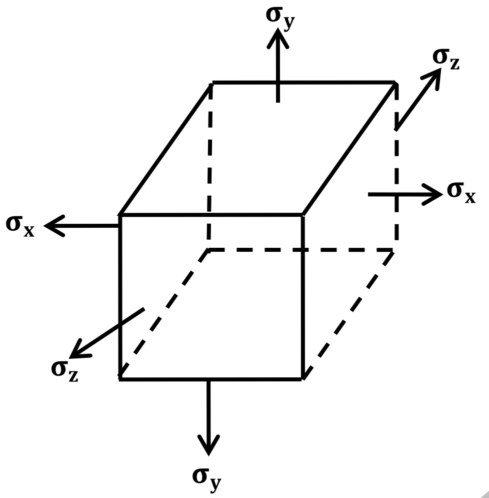
$\sigma_x = \sigma_y = \sigma_z = \sigma$

$\epsilon = \frac{(1-2v)}{E}.\sigma$

$\epsilon = -\propto\Delta T$ → thesmal compression strain.

$\sigma = -\frac{\propto\Delta T}{(1-2v)}E$

$\sigma = -\frac{\propto\Delta T}{(1-2v)}E$ → compressive stress.



43. Water ($C_p = 4.18$ kJ/kg.K) at 80°C enters a counter flow heat exchanger with a mass flow rate of 0.5 kg/s. Air ($C_p = 1$ kJ/kg.K) enters at 30°C with a mass flow rate of 2.09 kg/s. If the effectiveness of the heat exchanger is 0.8, the LMTD (in °C) is

(A) 40
(B) 20
(C) 10
(D) 5

[Ans. C]

$\varepsilon = \frac{C_h}{C_{min}} \times \frac{(t_{h_1}-t_{h_2})}{(t_{h_1}-t_{c_1})}$

$0.8 = \frac{4.18 \times 0.5}{2.09 \times 1} \times \frac{(80 - t_{h_2})}{(80 - 30)}$

$t_{h_2} = 40^oC$

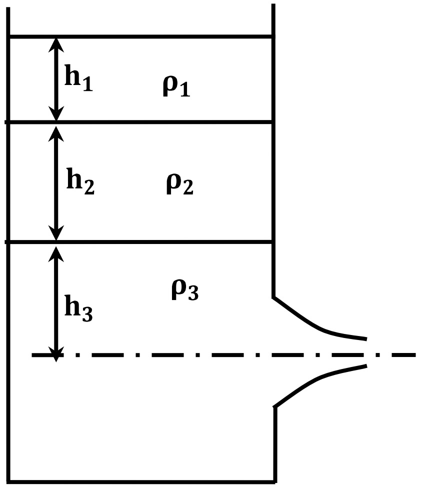
$m_h c_{p_h} (t_{h_1} - t_{h_2}) = m_c c_{p_c} (t_{c_2} - t_{c_1})$

$0.5 \times 4.18 \times 40 = 2.09 \times 1 \times (t_{c_2} - 30)$

$t_{c_2} = 70^oC$

$\theta_1 = \theta_2 = 10^oC \therefore \text{LMTD} = 10^oC$

44. A large tank with a nozzle attached contains three immiscible, inviscid fluids as shown. Assuming that the changes is h_1, h_2 and h_3 are negligible, the instantaneous discharge velocity is



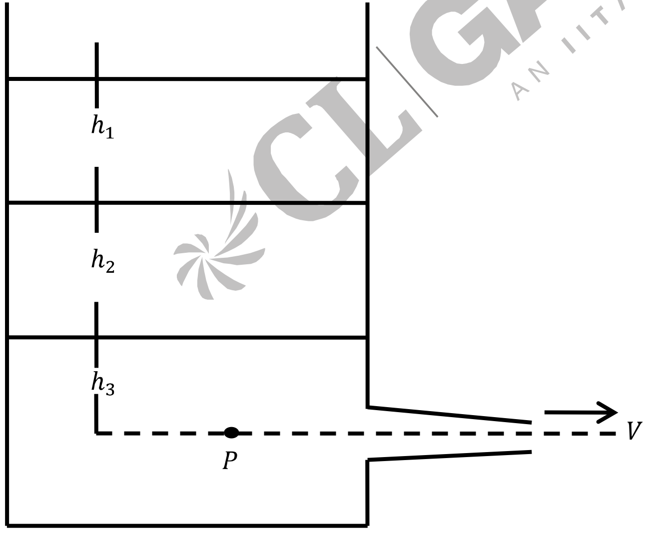
(A) $\sqrt{2gh_3\left(1+\frac{\rho_1}{\rho_3}\frac{h_1}{h_3}+\frac{\rho_2}{\rho_3}\frac{h_2}{h_3}\right)}$

(B) $\sqrt{2g\,(h_1+h_2+h_3)}$

(C) $\sqrt{2g\left(\frac{\rho_1 h_1+\rho_2 h_2+\rho_3 h_3}{\rho_1+\rho_2+\rho_3}\right)}$

(D) $\sqrt{2g\left(\frac{\rho_1 h_2 h_3+\rho_2 h_3 h_1+\rho_3 h_1 h_2}{\rho_1 h_1+\rho_2 h_2+\rho_3 h_3}\right)}$

[Ans. A]



At any point P, pressure is given by

$P = \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3$

By Bernoulli's theorem.

$$P = \frac{1}{2}\rho_3 V^2$$

$$\frac{1}{2}\rho_3 v^2 = \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3$$

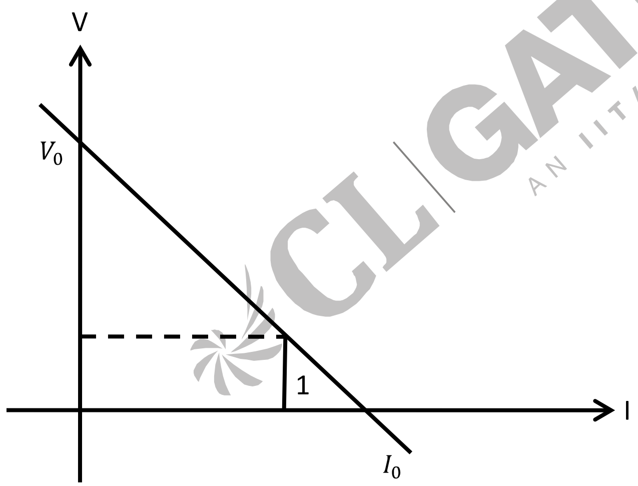
$$V^2 = 2gh_3\left(1 + \frac{\rho_1}{\rho_3}\frac{h_1}{h_3} + \frac{\rho_2}{\rho_3}\frac{h_2}{h_3}\right)$$

$$V = \sqrt{2gh_3\left(1 + \frac{\rho_1}{\rho_3}\frac{h_1}{h_3} + \frac{\rho_2}{\rho_3}\frac{h_2}{h_3}\right)}$$

45. In a DC arc welding operation, the voltage-arc length characteristic was obtained as $V_{arc} = 20 + 5l$ where the arc length l was varied between 5 *mm* and 7 *mm*. Here V_{arc} denotes the arc voltage in *Volts*. The *arc current* was varied from 400 A to 500 A. Assuming linear power source characteristic, the *open circuit* voltage and the *short circuit current* for the welding operation are

(A) 45 V, 450 A
(B) 75 V, 750 A
(C) 95 V, 950 A
(D) 150 V, 1500 A

[Ans. C]



$V_{arc} = 20 + 5l$

$5^m < l < 7mm$

400 A < I < 500A

For $l = 5mm \implies \quad V_{arc} = 20 + 5 \times 5 = 45V$

For $l = 7mm \implies V_{arc} = 20 + 5 \times 7 = 55V$

$$\frac{V}{V_0} + \frac{I}{I_0} = 1$$

For I = 400A , V = 55V (when current is less, voltage will be more)

$\frac{55}{V_0}+\frac{400}{I_0}=1$ (1)------------

For I = 500A, V = 45V

$\Rightarrow \frac{45}{V_0}+\frac{500}{I_0}=1$ ------(2)----

On solving

$V_0 = 95V$ $I_0 = 950$ V

46. In a single pass rolling process using 410 mm diameter steel rollers, a strip of width 140 mm and thickness 8 mm undergoes 10% reduction of thickness. The *angle of bite* in *radians* is

(A) 0.006
(B) 0.031
(C) 0.062
(D) 0.600

[Ans. C]

Angle of Bite $\theta=\sqrt{\frac{t_i-t_f}{R}}$

$=\sqrt{\frac{8-7.2}{205}}=0.062$

47. Calculate the punch size in mm, for a circular blanking operation for which details are given below.

Size of the blank	25 mm
Thickness of the sheet	2 mm
Radial clearance between punch and die	0.06 mm
Die allowance	0.05 mm

(A) 24.83
(B) 24.89
(C) 25.01
(D) 25.17

[Ans. A]

Punch size = Blank size – 2 C_r – C_d
= 25 – 2 × 0.06 – 0.05
= 24.83 mm

Common Data Questions

Common Data for Question 48 and 49:

A refrigerator operates between 120 kPa and 800 kPa in an ideal vapor compression cycle with R-134a as the refrigerant. The refrigerant enters the compressor as saturated vapor and leaves the condenser as saturated liquid. The mass flow rate of the refrigerant is 0.2 kg/s. Properties for R-134a are as follows:

Saturated R-134a

P (kPa)	T(°C)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg.K)	s_g (kJ/kg.K)
120	-22.32	22.5	237	0.093	0.95
800	31.31	95.5	267.3	0.354	0.918

Superheated R-134a

P (kPa)	T (°C)	h (kJ/kg)	s (kJ/kg.K)
800	40	276.45	0.95

48. The rate at which heat is extracted, in kJ/s from the refrigerated space is

(A) 28.3
(B) 42.9
(C) 34.4
(D) 14.6

[Ans. A]

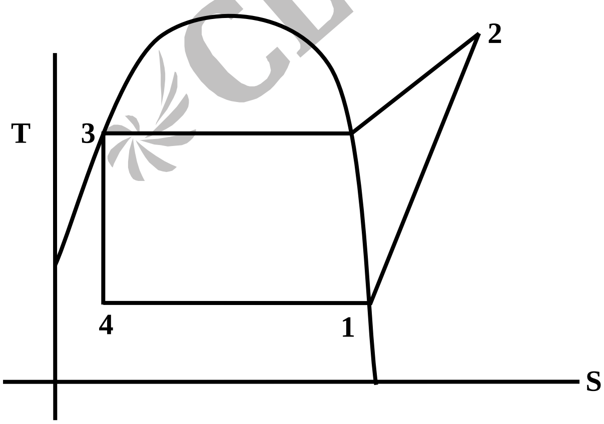
Rate at which heat is extracted = $m_r\,(h_1 - h_4)$

= 0.2 (237 – 95.5) = 28.3 kW

49. The power required for the compressor in kW is

(A) 5.94
(B) 1.83
(C) 7.9
(D) 39.5

[Ans. C]

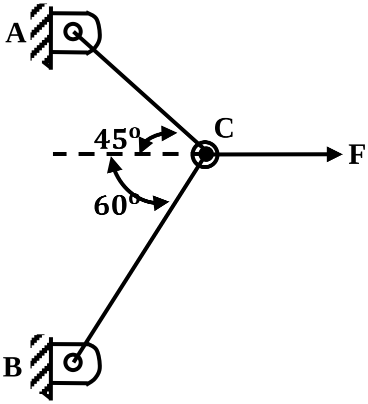


Power required for compressor = $m_r\,(h_2 - h_1)$

= 0.2 × (276.45 – 237) = 7.9 kW

Common Data for Questions 50 and 51:

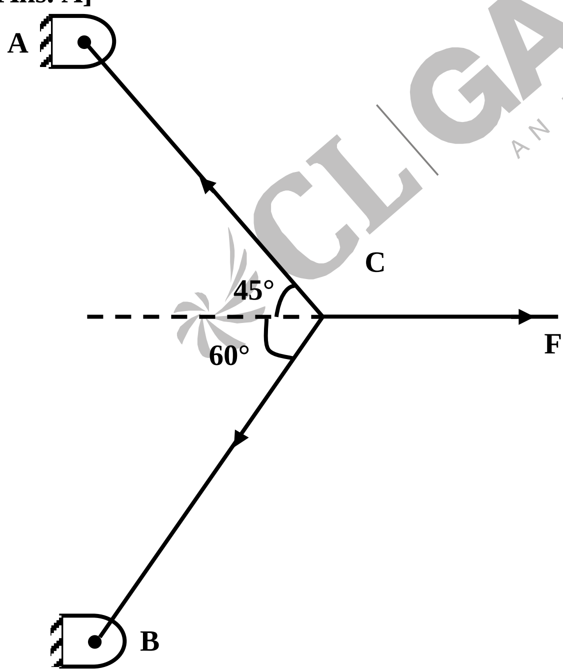
Two steel truss members, AC and BC, each having cross sectional area of 100 mm^2, are subjected to a horizontal force **F** as shown in figure. All the joints are hinged.



50. If **F** = 1 kN, the magnitude of the vertical reaction force developed at the point B in kN is

(A) 0.63
(B) 0.32
(C) 1.26
(D) 1.46

[Ans. A]



$R_{AC}\cos 45 + R_{BC}\cos 60 = F$

$\Rightarrow \frac{R_{AC}}{\sqrt{2}} + \frac{R_{BC}}{2} = F$ -------- (1)

$R_{AC} \sin 45 = R_{BC} \sin 60$

$\Rightarrow \frac{R_{AC}}{\sqrt{2}} = \frac{\sqrt{3}R_{BC}}{2}$

$\Rightarrow R_{AC} = \sqrt{6}.\frac{R_{BC}}{2}$ --------- (2)

From equation (1) and (2)

$R_{BC} = \frac{2F}{(\sqrt{3}+1)}$ $R_{AC} = \frac{\sqrt{6}\,F}{\sqrt{3}+1}$

$F = 1 \Rightarrow R_{BC} = \frac{2}{\sqrt{3}+1}$

Vertical Reaction $V_B = R_{BC} \cos 30$

$= \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}}{2}$

$V_B = 0.63$

51. The maximum force **F** in kN that can be applied at C such that the axial stress in any of the truss members DOES NOT exceed 100 MPa is

(A) 8.17
(B) 11.15
(C) 14.14
(D) 22.30

[Ans. B]

From sol. of 50

$R_{AC} = \frac{\sqrt{6}F}{\sqrt{3}+1}$ $R_{BC} = \frac{2F}{\sqrt{3}+1}$

Since $R_{AC} > R_{BC}$

$\therefore 100\ MPa = \frac{R_{AC}}{100}$

$100 \times 100 = \frac{\sqrt{6}\,F}{(\sqrt{3}+1)}$

$F = \frac{(\sqrt{3}+1)}{\sqrt{6}} \times 10\ KN$

$F = 11.15\ KN$

Linked Answer Questions

Statement for Linked Answer Question 52 and 53:

Air enters an adiabatic nozzle at 300 kPa, 500 K with a velocity of 10 m/s. It leaves the nozzle at 100 kPa with a velocity of 180 m/s. The inlet area is 80 cm^2. The specific heat of air C_p is 1008 J/kg.K.

52. The exit temperature of the air is

(A) 516 K
(B) 532 K
(C) 484 K
(D) 468 K

[Ans. C]

$h_1 + v_1^2/2 = h_2 + v_2^2/2$

$\frac{V_2^2 - V_1^2}{2} = h_1 - h_2 = C_P (T_1 - T_2)$

$\frac{180^2 - 10^2}{2} = 1008 \times (500 - T_2)$

$T_2 = 484k$

53. The exit area of the nozzle in cm^2 is

(A) 90.1
(B) 56.3
(C) 4.4
(D) 12.9

[Ans. D]

From conlunty of 2 inlet and outlet flow in nozzle.

ρAv = const. (mass conservation)

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$A_2 = \left(\frac{\rho_1}{\rho_2}\right) \times \left(\frac{v_1}{v_2}\right) \times A_1$

$A_1 \rightarrow$ inlet area of nozzle, $\rho_1 \rightarrow$ inlet density

$A_2 \rightarrow$ exit area of nozzle, $\rho_2 \rightarrow$ exit density

$A_1 = 80\ cm^2$ $\quad P_1 = 300KPa$ inlet pressure

$\rho = \frac{P}{RT}$ $\quad P_2 = 100KPa$, exit pressure

$A_2 = \left(\frac{P_1}{P_2}\right) \times \left(\frac{T_2}{T_1}\right) \times \left(\frac{V_1}{V_2}\right) \times A_1$

$A_2 = 3 \times \frac{484}{500} \times \frac{10}{180} \times 80 = 12.906\ cm^2$

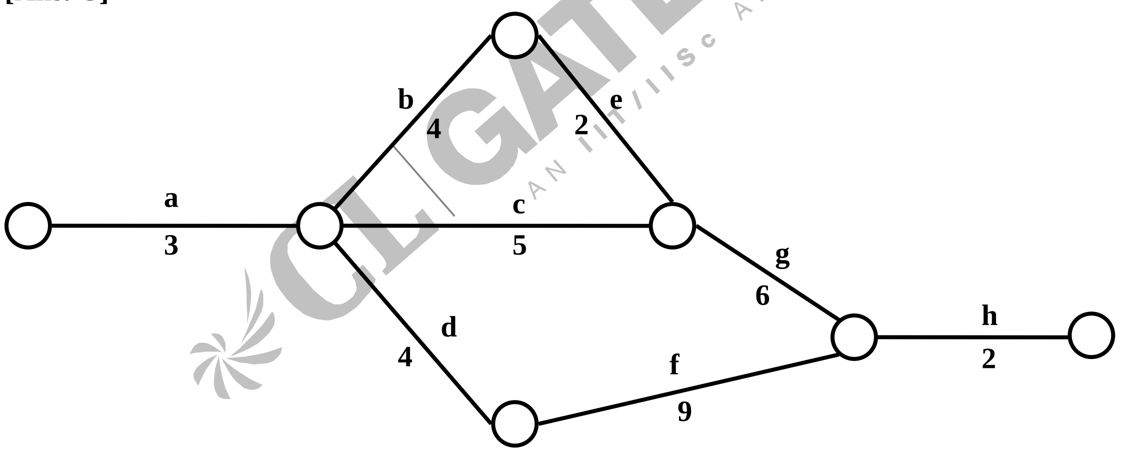
Statement for Linked Answer Questions 54 and 55:

For a particular project, eight activities are to be carried out. Their relationships with other activities and expected durations are mentioned in the table below.

Activity	Predecessors	Duration (days)
a	-	3
b	a	4
c	a	5
d	a	4
e	b	2
f	d	9
g	c, e	6
h	f, g	2

54. The critical path for the project is
(A) $a - b - e - g - h$
(B) $a - c - g - h$
(C) $a - d - f - h$
(D) $a - b - c - f - h$

[Ans. C]



Critical path for the project is
$a - d - f - h$
Total time = 18 days.

55. If the duration of activity f alone is changed from 9 to 10 days, then the
(A) Critical path remains the same and the total duration to complete project changes to 19 days.
(B) Critical path and the total duration to complete the project remain the same.
(C) Critical path changes but the total duration to complete the project remains the same.

(D) Critical path changes and the total duration to complete the project changes to 17 days.

[Ans. A]

Critical path remains same and total duration will increase by 1 day and it takes 19 days to complete.

General Aptitude (GA) Questions (Compulsory)

Q. 56 – Q. 60 carry one mark each.

56. Choose the grammatically **INCORRECT** sentence:
(A) They gave us the money back less the service charges of Three Hundred rupees.
(B) This country's expenditure is not less than that of Bangladesh.
(C) The committee initially asked for a funding of Fifty Lakh rupees, but later settled for a lesser sum.
(D) This country's expenditure on educational reforms is very less.

[Ans. D]

57. Choose the most appropriate alternative from the options given below to complete the following sentence:
Despite several _____________ the mission succeeded in its attempt to resolve the conflict.
(A) attempts
(B) setbacks
(C) meetings
(D) delegations

[Ans. B]

The word despite tells us that mission succeeded even though there were problems. Hence the correct answer is Setbacks

58. Choose the most appropriate alternative from the options given below to complete the following sentence.
Suresh's dog is the one __________ was hurt in the stampede.
(A) that
(B) which
(C) who
(D) whom

[Ans. A]

The correct answer is 'that'. In the given sentence 'was hurt in the stampede' determines which dog belongs to Suresh.

59. Which one of the following options is the closest in meaning to the word given below?

Mitigate

(A) Diminish
(B) Divulge
(C) Dedicate
(D) Denote

[Ans. A]

'Mitigate' means to make something less harmful, serious, intense etc. Hence correct answer is 'Diminish'.

60. The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of Rs 50 per unit. The number of units to be produced by the firm such that the profit is maximized is

(A) 5
(B) 10
(C) 15
(D) 25

[Ans. A]

Profit = $50q - 5q^2$, by putting the value of the options, we get the maximum profit at $q = 5$ units.

Alternate Method:

$P = 50q - 5q^2$, $P' = 50 - 10q$, for minimum or maximum value $50 - 10q = 0$, P" is negative. So, the maximum value of $q = 5$.

Q. 61 – Q. 65 carry two marks each.

61. Which of the following assertions are **CORRECT**?

P: Adding 7 to each entry in a list adds 7 to the mean of the list
Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list
R: Doubling each entry in a list doubles the mean of the list
S: Doubling each entry in a list leaves the standard deviation of the list unchanged

(A) P, Q
(B) Q, R
(C) P, R
(D) R, S

[Ans. C]

When each entry is increased or decreased by same value k, the mean will also increase or decrease by k but the standard deviation will remain same. But when each entry is multiplied or divided by same value k, the mean as well as the standard deviation will get multiplied by k.

62. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1\,x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(A) 8 meters
(B) 10 meters
(C) 12 meters
(D) 14 meters

[Ans. B]

$y = 2x - 0.1x^2$ or $y' = 2 - 0.2x$ or for the maximum or minimum value

$2 - 0.2x = 0$ or the value of x is 10. y” is negative. So the maximum value of y is 10.

63. Given the sequence of terms, AD CG FK JP, the next term is

(A) OV
(B) OW
(C) PV
(D) PW

[Ans. A]

The first letter increases by 2, 3 and 4 letters and the second letter increases by 3, 4 and 5 letters. Hence J will increase by 5 letters and P will increase by 6 letters. So the answer will be OV.

64. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X’s shock absorbers, 96% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is

(A) 0.288
(B) 0.334
(C) 0.667
(D) 0.720

[Ans. B]

Let total number of shock absorbers be a. Since 60% shock absorbers are supplied by X and 96% of them are reliable. Number of reliable shock absorbers supplied by X = 96% of 60% of a = 0.576a

Similarly, number of reliable shock absorbers supplied by Y = 72% of 40% of a = 0.288a

Total number of reliable shock absorbers = 0.576a + 0.288a = 0.864a

Probability that a reliable shock absorber is made by Y = (0.288a / 0.864a)

= 0.334

65. **Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed.**
Which one of the following is the best inference from the above advertisement?
(A) Gender-discriminatory
(B) Xenophobic
(C) Not designed to make the post attractive
(D) Not gender-discriminatory

[Ans. D]
Part time job with post attractive because transportation paid and expenses reimbursed. So, there is no gender-discriminatory in this part time job.